RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. SECOND SEMESTER (January – June) 2014

Mid-Semester Examination, March 2014

Date : 24/03/2014 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : II Full Marks : 50

[Use a separate Answer Book for each group]

Group - A

Answer **any one** question : [1×6]

1. a) If a, b, c be positive real numbers and abc = K^3, prove that $(1+a)(1+b)(1+c) \geq (1+K)^3$. [3]

   b) If z be a complex number with modulus 1 and amplitude $\theta (0 < \theta < \pi)$ find the modulus and the principal amplitude of $\frac{1-z}{1+z}$. [3]

2. a) Z is a variable complex number such that $\left|\frac{z-i}{z+i}\right| = K$. Show that the point z lies on a circle in the complex plane if $K \neq 1$ and z lies on a straight line if K = 1. [3]

   b) Prove that the minimum value of $x^2 + y^2 + z^2$ is $\left(\frac{c}{7}\right)^2$ where x,y,z are positive real numbers subject to the condition 2x+3y+6z = c, c being a constant. [3]

Answer **any one** question : [1×6]

3. If α is a multiple root of the equation $f(x) = 0$ of order r then show that α is a multiple root of $f'(x) = 0$ of order r–1.
   Show that the equation $x^4 - 14x^2 + 24x + K = 0$ has four real and unequal roots if $-11 < K < -8$. [2+4]

4. State Descartes' rule of sign.
   Apply Descartes' rule of sign to determine the nature of the roots of $x^6 - 3x^2 - 2x - 3 = 0$. [2+4]

Answer **any two** questions : [2×5]

5. State Bolzano's Weiertrass theorem on [a,b] for a continuous function.
   Let $f:[0,1] \to [0,1]$ be a continuous function. Does there exist a point $c \in [0,1]$ such that f(c) = c? Verify. [2+3]

6. a) State Intermediate value theorem for a continuous function on [a,b].

   b) Show that $f(x) = x^3 + 4x + 4$ has a real root.

   c) Give an example of a function which is not continuous but have intermediate value property. [1+2+2]

7. a) Apply intermediate value property to show that the equation $\sqrt{x^6 + 5x^4 + 9} = 3 \cdot 6$ has a solution in the interval [0,1].

   b) Show that the set $\{x \in \mathbb{R} \mid x^3 + x^2 + 3x + 9 = 5x^4\}$ is a closed subset of $\mathbb{R}$.

   c) Show that the set $\{x \in \mathbb{R} \mid f(x) \neq 0\}$ contains both rational and irrational point if $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. [1+2+2]

Answer **any one** questions : [1×3]

8. Determine whether the following functions are continuous. If they not, determine where their discontinuities are and classify them.

a) $f(x)=\begin{cases}2x+4, x<0\\ -x-3, x\geq 0\end{cases}$

b) $f(x)=\begin{cases}x^2+3 & x\neq 0\\ 4 & x=0\end{cases}$ [2+1]

9. a) Is $\sin^{-1}x$ is continuous on the real numbers?
   b) Show that no continuous function from [a,b] to $[a,b)$ is onto. [1+2]

Group - B

Answer **any two** questions : [2×5]

10. Prove without expanding that

$$\begin{vmatrix}a^4 & a^2 & a & 1\\ b^4 & b^2 & b & 1\\ c^4 & c^2 & c & 1\\ d^4 & d^2 & d & 1\end{vmatrix}=(a+b+c+d)\begin{vmatrix}a^3 & a^2 & a & 1\\ b^3 & b^2 & b & 1\\ c^3 & c^2 & c & 1\\ d^3 & d^2 & d & 1\end{vmatrix}$$

11. If A be an invertible matrix then show that A^t is invertible and $(A^t)^{-1}=(A^{-1})^t$.

12. If A is a non-singular matrix such that the sum of the elements in each row is K, prove that the sum of the elements in each row of A^{-1} is K^{-1}.

13. If A is a real orthogonal matrix and I+A is non-singular, prove that the matrix $(I+A)^{-1}(I-A)$ is skew-symmetric.

Answer **any three** questions : [3×5]

14. Reduce the feasible solution $x_1=2,\ x_2=1,\ x_3=1$ to the system of equations

$x_1+4x_2-x_3=5$

$2x_1+3x_2+x_3=8$ to B.F.S.

15. Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b, $x\geq 0$ corresponds to a B.F.S.

16. a) Prove that of all convex combinations of a finite number of points is a convex set.
    b) Show that $X=\{x:|x|\leq 2\}$ is a convex set.

17. Solve by Simplex Method :

Maximize $z=5x_1+3x_2$

Subject to $3x_1+5x_2\leq 15$

$5x_1+2x_2\leq 10$

$x_1, x_2\geq 0$

18. Find all the basic solutions of the following equations identifying in each case the basic vectors and the basic variables :

$x_1+x_2+x_3=4$

$2x_1+5x_2-2x_3=3$